

THE BEHAVIOR OF PCM/PM RECEIVERS IN NON-IDEAL CHANNELS

Part H: The Combined Effect of Imperfect Data Streams and Band-limiting Channels on Performance¹

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ABSTRACT

This paper presents a performance evaluation of residual carrier communication systems that employ a PCM/PM modulation technique operating over non-ideal channels. The non-ideal channels under investigation include data asymmetry, an unbalanced data stream (i. e., transition density deviates from 0.5) and InterSymbol Interference (ISI). In this particular modulation scheme, the data (either NRZ or Bi- ϕ) is directly modulated on the RF residual carrier. The combined effect of both an imperfect data stream (e.g., data asymmetry and an unbalanced data stream) and ISI on the average Symbol Error Rate (SER) is determined for NRZ and Bi- ϕ data formats, and the results are compared. The performance degradations for uncoded and coded systems are evaluated and compared. For coded systems, the performance degradation is evaluated for convolutional code with rate 1/2 and constraint length 7.

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1. Introduction

In part I of this paper [1], the separate effects of data asymmetry, unbalanced data stream and bandlimiting data channels on the performance degradations of the space telemetry systems that employed PCM/PM modulation scheme have been investigated. [1] studied in detailed the effects of each of the undesired sources such as data asymmetry, unbalanced data and ISI caused by bandlimiting channel on the Symbol Error Rate (SER) performance of the PCM/PM receivers. Moreover, the performance of the PCM/PM with NRZ data format (PCM/PM/NRZ) has also been compared with PCM/PM/Bi- ϕ in [1].

Since in reality, the practical PCM/PM receivers operate in the presence of both data imperfections and bandlimiting channel, Note that the term "imperfect data stream" considered in this paper includes the effects of both data asymmetry and imbalance between +1's and -1's in the data stream (namely unbalanced data stream). Theoretical predictions for the total symbol SNR degradation of the receivers due to the presence of these three undesired sources are not the algebraic sum of each symbol SNR degradation due to a single source of degradation found in [1]. Therefore, it is necessary to study the combined effects of these three sources on the error probability performance. In this paper, mathematical models to evaluate the SER performances of the PCM/PM/NRZ and PCM/PM/Bi- ϕ receivers in the presence of both imperfect data stream and bandlimiting channel will be derived,

In the past, [3] has analyzed the combined effects of both data asymmetry and bandlimiting channel on the performance of the suppressed carrier systems where the carrier tracking is not disturbed by the data interferences. Furthermore, when analyzed the combined effects of data asymmetry and ISI caused by bandlimiting channel, [3] assumed that the amount of data asymmetry is known so that an optimum sampling time can be set for the sample detector.

The aim of this paper is to investigate and assess the impacts of the combined effects of unbalanced data stream, data asymmetry and ISI on the performance degradation of the PCM/PM receivers. Both data formats, NRZ and Bi- ϕ , are considered. This extends previously reported work [3] to include PCM/PM modulation schemes and the presence of unbalanced data stream on the transmitting signal.

This paper is organized as follows: Section 2 introduces briefly the space telemetry system models employing PCM/PM modulation technique and SER performance for ideal operating conditions. Section 3 derives a mathematical model to evaluate the combined effects of unbalanced data stream, data asymmetry and ISI on the PCM/PM/NRZ system performance. The combined effects of both imperfect data stream and bandlimiting data channel on PCM/PM/Bi- ϕ are analyzed in Section 4. Performance comparisons for PCM/PM/NRZ and PCM/PM/Bi- ϕ of both uncoded and coded systems are presented in Section 5. Finally, Section 6 presents the main conclusion of the paper.

2. Space Telemetry System Models

Atypical space telemetry system model shown in Figure 1 of [1] will be considered in this paper. Again, In this mode], the data stream can be either NRZ. or Bi- ϕ (Manchester or Bi-phase) data stream with $\xi\%$ data asymmetry and a transition density, p_t , which is less than 1/2. The data symmetry models for NRZ and Bi- ϕ data streams use in the following analyses will be the same as shown in Figures 4(a) and 4(b) of [1], respectively. The mathematical model the transmitted telemetry signal is given by

$$S_T(t) = \sqrt{2P} \cos(\omega_c t + m_T d(t)) \quad (1)$$

where P is the transmitted power, $\omega_c = 2\pi f_c$ is the angular carrier center frequency in rad/sec, m_T is the telemetry modulation index in rads which is less than $\pi/2$, and $d(t)$ is NRZ data Sequence (PCM/PM/NRZ) or the Manchester data waveform generated by the binary (± 1) NRZ data sequence (PCM/PM/Bi- ϕ).

The received signal $S_r(t)$ is corrupted by additive white Gaussian noise $n(t)$ with one-sided noise spectral density N_0 , data asymmetry, unbalanced data and ISI. Expanding the received signal we have

$$S_r(t) = \sqrt{2P} \left[\cos(m_T) \cos(\omega_c t + \theta_o) - d(t) \sin(m_T) \sin(\omega_c t + \theta_o) \right] + n(t) \quad (2)$$

where ω_o is the initial phase offset caused by the transmission medium. The first and second terms of Equation (2) are the residual carrier and data components, respectively.

As explained previously in [1, 2 and 4], the data asymmetry and imbalance between +1's and -1's in the data stream will produce undesired spectral components at the carrier frequency creating an imperfect carrier reference which will in turn degrade the telemetry system performance. In addition, the presence of ISI created by the band-limited channel can cause further disturbance to the carrier reference.

Let θ_e be the phase error due to the thermal noise and the interference caused by the data asymmetry and unbalanced data stream then one can show that the signal output of the integrate-and-dump at time $t = T_s$ (where T_s denotes the symbol period) is given by [1]

$$Z(T_s) = \sqrt{P} \sin(m_T) \cos(\theta_e) \int_0^{T_s} d(t) C(t) dt + n(T_s) \quad (3)$$

where $C(t)$ is the symbol sync clock shown in Figures 3(a) and 3(b) of [1] for NRZ and Bi- ϕ , respectively, Eqn (3) assumes that the phase error process θ_e is essentially constant during the symbol interval T_s , and that the corrupting noise process $n(T_s)$ is a zero-mean Gaussian random variable with a variance $N_0 T_s/2$.

The test statistic $Z(T_s)$ of Eqn (3) represents the observed data at the receiver. This test statistic is needed to determine the SER performance. Based on this test statistic, the performance of the telemetry system shown in Figure 1 has been evaluated in [1] for separate undesired sources of degradation (e.g., data asymmetry, unbalanced data and ISI). On the other hand, [5] has derived the SER for perfect data stream and unlimited bandwidth channel. The results are presented here for the purpose of comparison,

The average probability of error is given by [1]

$$P_e = \int_{\theta_e} P_e(\theta_e) P(\theta_e) d\theta_e \quad (4)$$

where $P_e(\theta_e)$ is the conditional probability of error and $P(\theta_e)$ is the probability density function (pdf) for θ_e . For perfect data stream and ideal channel, this conditional probability of error is given by [5]:

$$P_e(\theta_e) = (1/2) \operatorname{erfc}\{\sqrt{E_s/N_o} \cos(\theta_e)\} \quad (5)$$

where E_s denotes the symbol energy, i.e., $E_s = (PT_s)\sin^2(m_T)$. In this paper, one also postulates a Tikhonov pdf for θ_e , which is entirely characterized by the variance σ^2 of the carrier tracking phase error. When the loop signal-to-noise ratio is high the Tikhonov pdf can be approximated by

$$P(\theta_e) \approx \exp(-\theta_e^2/2\sigma^2)/[2\pi\sigma^2]^{1/2}, \quad -\infty < \theta_e < \infty \quad (6)$$

For perfect data stream and high-data-rate case ($B_L/R_s \ll 0.1$, where B_L and R_s denote the one-sided loop bandwidth and the symbol rate, respectively), the variance of the carrier tracking phase error, σ^2 , has been found in [5]. For perfect NRZ data format, it is given by

$$\sigma^2 = (1/p_o) + (B_L/R_s)\tan^2(m_T) \quad (7)$$

and, for perfect Bi- ϕ data format, σ^2 becomes

$$\sigma^2 = (1/p_o) + (I/C)\tan^2(m_T) \quad (8)$$

where

$$p_o = \frac{(E_s/N_o)}{(B_L/R_s)\tan^2(m_T)}, \quad (9)$$

$$I/C = (1/2) + (9/16)(B_L/R_s)^{-1} \\ - (3/4)(B_L/R_s)^{-1} \exp\{-(2/3)(B_L/R_s)\} [\cos\{(2/3)(B_L/R_s)\} + 3\sin\{(2/3)(B_L/R_s)\}]$$

$$\begin{aligned}
& + (3/16)(B_L/R_s)^{-1} \exp\{-(4/3)(B_L/R_s)\} [\cos\{(4/3)(B_L/R_s)\} \\
& + 3\sin\{(4/3)(B_L/R_s)\}]
\end{aligned} \tag{10}$$

in the following sections one will determine the conditional error probability and the carrier tracking phase error when the data stream is disturbed by the data asymmetry, unbalanced data stream and ISI caused by bandlimiting channel.

3. Combined Effects on PCM/PM/NRZ Receivers

To determine the average SER in the presence of data asymmetry, unbalanced data stream and bandlimiting channel one will use the same approach presented in [1], i.e., we will find the probability of error conditioned on the carrier tracking phase error and the distribution of the phase error. Here the distribution of the phase error will be determined first.

Since one postulates a Tikhonov distribution for the phase error θ_e , the variance of θ_e completely characterizes the pdf of the carrier tracking phase error. Again using the linear model for the carrier tracking loop one can evaluate modified noise spectral density, N , resulting from the thermal noise, data asymmetry and unbalanced data [1, Eqn (16)].

To evaluate N one must derive the data power spectrum of an unbalanced and asymmetric NRZ data stream. Based on the asymmetric and unbalanced data stream shown in Figure 4(a), [2] has derived the power spectral density for the asymmetric NRZ data stream generated by a purely random source with a transition density P , less than 1/2 (i.e. unbalanced data stream). The continuous spectrum component ($SC(f)$), the dc ($S_{dc}(f)$) and harmonics ($S_h(f)$) components of the asymmetric and unbalanced NRZ data stream are given by [2]

$$\begin{aligned}
S_{cNRZ}(f) = & T_s [\sin(\pi f T_s) / (\pi f T_s)]^2 [a_1(p_i) + a_2(p, p_i, \xi)] + T_s a_3(p, \xi) [\sin(\pi f T_s \xi) / (\pi f T_s)]^2 \\
& + T_s [\sin(2\pi f T_s) / (\pi f T_s)^2] [a_4(p, p_i, \xi) - a_5(p, p_i)]
\end{aligned} \tag{11}$$

$$S_{dcNRZ}(f) = [2p(1 - 2\xi p_i)]^2 \delta(f) \tag{12}$$

$$S_{hNRZ}(f) = 2(P_i/\pi)^2 \sum_{m=1}^{\infty} (1/m^2) c(m, p, \xi) \delta(f - m R_s) \tag{13}$$

where

$$a_1(p_i) = p_i(1 - p_i)[1 + 2(1 - p_i)] = p_i^3 \tag{14}$$

$$a_2(p, p_i, \xi) = \{3p_i^3 + p_i(1 - p_i)[1 + 2(1 - 2p)]\} \cos^2(\pi f T_s \xi) \tag{15}$$

$$a_3(p, \xi) = p_i(1 + p_i^2 - p_i) \cos^2(\pi f T_s) + p_i^3 \cos(2\pi f T_s \xi) \tag{16}$$

$$a_4(p, p_i, \xi) = p_i(1 - p_i)(1 - 2p)[0.5 \cos(2\pi f T_s \xi) - p \sin(2\pi f T_s \xi)] \tag{17}$$

$$a_s(p, p_t) = 0.5p_t(1 - p_t)(1 - 2p) \quad (18)$$

$$c(m, p, \xi) = \sin^2(m\pi\xi)[\cos^2(m\pi\xi) - (1 - 2p)^2\sin^2(m\pi\xi)] \quad (19)$$

Where p and p_t are defined as the probability of transmitting a + 1 pulse and transition density, respectively. For a purely random data source, the transition density is given by $P_t = 2p(1 - p)$ (20)

Note that when $p = p_t = 1/2$, i.e., the data stream is balanced, Eqns (11)-(13) reduce to Eqns (21)-(22) of [1]; and when $p = p_t = 1/2$ and $\xi = 0$, i.e., perfect data stream, Eqns (11)-(13) reduce to the well-known result for a perfect NRZ random data stream [6].

From Eqns (19) and (20) of [1] and Eqns (11)-(12) above, one obtains the interference due to continuous spectrum component, α , and interference caused by dc component-to-carrier power ratio I/C , respectively. Consequently, the variance of the carrier tracking phase error can be obtained by using Eqn (18) of [1]. It is found to be

$$\sigma^2 = I/p_o + (\alpha/2)\tan^2(m_T) - (1/2) [2p - (1 - 2\xi p_t)]^2 \tan^2(m_t) \quad (21)$$

where

$$\alpha = \int_{-\infty}^{\infty} |H(2\pi f)|^2 S_{cNRZ}(f) df \quad (22)$$

The harmonic components caused by the asymmetry do not interfere with carrier tracking because we have assumed that $2B_L C < R_s$. Having determined the distribution of the carrier tracking phase error, the error probability conditioned on the phase error, $P_e(\theta_e)$ can be determined. When a sample detector shown in Figure 1 operates in the presence of data asymmetry and bandlimiting channel, the combined effects of these two sources of degradation have been investigated in [3]. However, the approach used in [3] is not applicable here because it is assumed that the amount of data is known and the optimum sampling time is not T_s . Here we are interested in the performance degradation of the receiver when the sampling time is T_s and the amount of data asymmetry is unknown.

This paper uses slightly different approach than [3] by assuming that the optimum sampling time is T_s . Based on the data asymmetry model presented in Figure 4(a) of [1], one has a set of signals contain four different symbols, namely, $\{P_{iNRZ}(t), i = 1, 2, 3, 4\}$, with associated probability $\{p_i; i = 1, 2, 3, 4\}$. We have

$$P_{iNRZ}(t) = \begin{cases} +1 & -(T_s/2) < t \leq (T_s/2)(1 + A) \\ 0 & \text{elsewhere} \end{cases} \quad (23)$$

$$P_{2 \text{ NRZ}}(t) = \begin{cases} -1 & -(T_s/2) < t \leq (T_s/2)(1 - A) \\ 0 & \text{elsewhere} \end{cases} \quad (24)$$

$$P_{3 \text{ NRZ}}(t) = \begin{cases} +1 & -(T_s/2) < t \leq (T_s/2) \\ 0 & \text{elsewhere} \end{cases} \quad (25)$$

$$P_{4 \text{ NRZ}}(t) = \begin{cases} -1 & -(T_s/2) < t \leq (T_s/2) \\ 0 & \text{elsewhere} \end{cases} \quad (26)$$

$$p_1 = \Pr\{g_{i \text{ NRZ}}(t) = g_{1 \text{ NRZ}}(t)\} = P \quad (27)$$

$$p_2 = \Pr\{g_{i \text{ NRZ}}(t) = g_{2 \text{ NRZ}}(t)\} = (1 - p) \quad (28)$$

$$p_3 = \Pr\{g_{i \text{ NRZ}}(t) = g_{3 \text{ NRZ}}(t)\} = p(1 - P) \quad (29)$$

$$p_4 = \Pr\{g_{i \text{ NRZ}}(t) = g_{4 \text{ NRZ}}(t)\} = (1 - p)(1 - P) \quad (30)$$

where the data asymmetry is defined as

$$\xi = \Delta/2 \quad (31)$$

For ideal bandpass filter (Eqn (53), [1]) with imperfect data stream, the output of the filter $g_{i \text{ NRZ}}(t)$ corresponding to the input $P_{i \text{ NRZ}}(t)$ can be obtained by substituting Eqn (53) into Eqn (45) of [1] with $P(t)$ is replaced by $P_{i \text{ NRZ}}(t)$. For **NRZ** data format, $g_{i \text{ NRZ}}(t + kT_s)$, for $i = 1, 2, 3, 4$, can be shown to be

$$g_{1 \text{ NRZ}}(t + kT_s) = \frac{1}{\pi} [\text{si}\{2\pi B(t + T_s(k + 1/2))\} - \text{si}\{2\pi B(t + T_s(k - 1/2 - \xi))\}] \quad (32)$$

$$g_{2 \text{ NRZ}}(t + kT_s) = -\frac{1}{\pi} [\text{si}\{2\pi B(t + T_s(k + 1/2))\} - \text{si}\{2\pi B(t + T_s(k - 1/2 + \xi))\}] \quad (33)$$

$$g_{3 \text{ NRZ}}(t + kT_s) = \frac{1}{\pi} [\text{si}\{2\pi B(t + T_s(k + 1/2))\} - \text{si}\{2\pi B(t + T_s(k - 1/2))\}] \quad (34)$$

$$g_{4 \text{ NRZ}}(t + kT_s) = -g_{3 \text{ NRZ}}(t + kT_s) \quad (35)$$

Note that the symbols $g_{i \text{ NRZ}}(t)$, $i = 1, 2, 3, 4$, are independent because d_k 's are independent.

Using the test statistic shown in Eqn (3) one can show that the conditional error probability of the PCM/PM/NRZ receiver in the presence of the bandlimiting channel and imperfect data channel is

$$\overline{P_e(\theta_e)} = p\Pr\{Z(T_s) < 0/\theta_e, d_0 = +1\} + q\Pr\{Z(T_s) > 0/\theta_e, d_0 = -1\} \quad (36)$$

where $q = (1 - p)$, and the overbar denotes statistical averaging over the joint distribution of the doubly infinite data sequence d_k and the test statistic $Z(T_s)$ for this particular case becomes:

$$Z(T_s) = E_s \left[\pm 1 + \sum_{k=-\infty}^{\infty} d_k \lambda_k(i) \right] \cos \theta_e + n(T_s) \quad (37)$$

where ± 1 corresponds to $d_0 = \pm 1$, and the prime in the sum indicates the omission of the term $k = 0$, and the parameter $\lambda_k(i)$ is defined as

$$\lambda_k(i) = \frac{\int_0^{T_s} g_{\text{NRZ}}(t) g_{\text{NRZ}}(t + kT_s) dt}{\int_0^{T_s} |g_{\text{NRZ}}(t)|^2 dt}, \quad i = 1, 2, 3, 4 \quad (38)$$

where $g_{\text{NRZ}}(t)$ is defined in Eqns (32)-(35) for $i = 1, 2, 3, 4$, respectively and $g_{\text{NRZ}}(t)$ is the output of the ideal filter for perfect data stream which is given by [Eqn (54), 1]. Note that in this case we do not have the symmetry of the signals (because the data under consideration is imperfect) hence we must take all possible combinations into account when computing Eqn (36). In order to illustrate the use of Eqn (36), an example will be provided for the case $M = 1$. For $M = 1$, Eqn (36) becomes

$$\begin{aligned} P_e(\theta_e) = & p[p_3 p_3 \text{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{-1}(3) + \lambda_{+1}(3))\cos\theta_e\} \\ & + p_3 p_1 \text{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{-1}(3) - \lambda_{+1}(1))\cos\theta_e\} \\ & + p_2 p_3 \text{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{-1}(2) + \lambda_{+1}(3))\cos\theta_e\} \\ & + p_2 p_1 \text{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{-1}(2) - \lambda_{+1}(1))\cos\theta_e\}] \\ & + q[p_1 p_2 \text{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{-1}(2) - \lambda_{+1}(2))\cos\theta_e\} \\ & + p_1 p_4 \text{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{-1}(1) + \lambda_{+1}(4))\cos\theta_e\} \\ & + p_4 p_2 \text{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{-1}(4) - \lambda_{+1}(2))\cos\theta_e\} \\ & - p_4 p_4 \text{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{-1}(4) + \lambda_{+1}(4))\cos\theta_e\}] \end{aligned} \quad (39)$$

The conditional error probability for $M = 2$ is shown in the appendix. The average probability of error can be found by substituting Eqn (36) and the pdf for the phase error just found above into Eqn (4) and performing the numerical integration in digital computer. The results of the calculations are shown in Figures 1-3 for the second order Phased-Locked Loop (PLL) with the transfer function given by Eqn (24) of [1].

Figure 1 plots the SER as a function of symbol SNR in dB for fixed data asymmetry ξ of 2 % and $BT_s = 3$ with p , probability of mark, is a parameter. This figure indicates that, for $m_T = 1.25$ rad, $2B_L/R_s = 0.001$, the SER degrades seriously as p deviates from 0.45. As mentioned in [1], the typical values for $m_T = 1.25$ rad and $2B_L/R_s = 0.001$ are chosen because the performance of PCM/PM approaches the ideal BPSK.

Figure 2 shows the SER performance for various values of data asymmetry with $BT_s = 3$ and $p = 0.45$. The symbol SNR degradation is less than 0.5 dB for $\xi = 6$ %. Furthermore, this figure shows that PCM/PM/NRZ is not sensitive to data asymmetry, because the symbol SNR degradation is between 0.1-0.2 dB when ξ varies between 2-6 %.

Figure 3 illustrates the SER performance in the presence of bandlimiting channel. The figure plots SER as a function of symbol SNR for $\xi = 2$ %, $p = 0.45$ with BT_s is a parameter. The results show that, for $BT_s = 3$, the symbol SNR degradation is at the order of 0.4 dB or less when the SER is between 10^{-7} - 10^{-9} .

4. Combined Effects on PCM/PM/Bi- ϕ Receivers

Using the same approach as shown in Section 3, one proceeds with the derivation of the power spectral density for the asymmetric Bi- ϕ data stream generated by a purely random NRZ source with a transition density P , less than 1/2. Based on the model of asymmetric and unbalanced data stream shown in Figure 4(b) of [1] one has a set of signals contain four different symbols, namely, $\{P_{iBi-\phi}(t), i = 1, 2, 3, 4\}$, with associated probability $\{p'_i; i = 1, 2, 3, 4\}$. One has

$$P_{1Bi-\phi}(t) = \begin{cases} +1 & -(T_s/2) \leq t \leq (\Delta T_s/4) \\ -1 & (\Delta T_s/4) < t \leq (T_s/2)(1 + A/2) \\ 0 & \text{elsewhere} \end{cases} \quad (40)$$

$$P_{2Bi-\phi}(t) = \begin{cases} -1 & -(T_s/2) \leq t \leq -(\Delta T_s/4) \\ 1 & -(\Delta T_s/4) < t \leq (T_s/2)(1 - A/2) \\ 0 & \text{elsewhere} \end{cases} \quad (41)$$

$$P_{3Bi-\phi}(t) = \begin{cases} +1 & -(T_s/2) \leq t \leq (\Delta T_s/4) \\ -1 & (\Delta T_s/2) < t \leq (T_s/2) \\ 0 & \text{elsewhere} \end{cases} \quad (42)$$

$$P_{4Bi-\phi}(t) = \begin{cases} -1 & -(T_s/2) \leq t \leq -(\Delta T_s/4) \\ +1 & -(\Delta T_s/4) < t \leq (T_s/2) \\ 0 & \text{elsewhere} \end{cases} \quad (43)$$

$$p'_1 = \Pr\{g_{iBi-\phi}(t) = g_{1Bi-\phi}(t) = p\}p_i \quad (44)$$

$$p'_2 = \Pr\{g_{iBi-\phi}(t) = g_{2Bi-\phi}(t) = (1-p)\}p_i \quad (45)$$

$$p'_3 = \Pr\{g_{iBi-\phi}(t) = g_{3Bi-\phi}(t) = p(1-p)\} \quad (46)$$

$$p'_4 = \Pr\{g_{iBi-\phi}(t) = g_{4Bi-\phi}(t)\} = (1-p)(1-p_i) \quad (47)$$

where the data asymmetry for Bi- ϕ is defined as

$$\xi = A/4 \quad (48)$$

It should be mentioned that the power spectral density for a purely random Bi- ϕ data with perfectly balanced data stream has been derived in [4]. Using the same technique presented in [4] and together with Eqns (40)-(47) we can show that the power spectral density for an asymmetric and unbalanced data stream illustrated in [Figure 4(b), 1] have the following form

$$S_{Bi-\phi}(f) = S_{cBi-\phi}(f) + S_{dcBi-\phi}(f) + S_{hBi-\phi}(f) \quad (49)$$

where $S_{cBi-\phi}(f)$, $S_{dcBi-\phi}(f)$ and $S_{hBi-\phi}$ are the continuous spectrum, dc and harmonic components of the imperfect Bi- ϕ data stream, respectively. They are given by

$$\begin{aligned} S_{cBi-\phi}(f) = & T_s p_i (1-p_i) [\sin^2(\pi f T_s/2) / (\pi f T_s/2)]^2 \\ & - T_s [c_1(p, \xi) + c_2(p, \xi) + c_3(p_i)] [\sin(\pi \xi f T_s) / (\pi f T_s/2)]^2 \\ & + T_s [c_4(p, p_i, \xi) + c_5(p, \xi)] [\sin(\pi \xi f T_s/2)]^2 \\ & - T_s c_6(p, p_i, \xi) [\sin(\pi \xi f T_s/2) / (\pi f T_s/2)]^2 \\ & + T_s c_7(p, p_i, \xi) [\sin(\pi f T_s \{1 + \xi\}/2) / (\pi f T_s/2)]^2 \\ & + T_s c_8(p, p_i, \xi) [\sin(\pi f T_s \{1 - \xi\}/2) / (\pi f T_s/2)]^2 \end{aligned} \quad (50)$$

$$S_{\text{dcBi-}\phi}(f) = \xi^2(2 - p_s) \delta(f) \quad (51)$$

$$S_{\text{hBi-}\phi}(f) = (2/\pi)^2 \sum_{m=1}^{\infty} (1/m^2) [H_1(m, p, p_s, \xi) + H_2(m, p, p_s, \xi) + H_3(m, p, p_s, \xi)] \delta(f - mR_s) \quad (52)$$

where

$$c_1(p_s, \xi) = 2p_s^2 \sin^2[\pi f T_s(1 + \xi)/2] [p_s \sin^2[\pi f T_s(1 - \xi)/2] + (1 - p_s) \cos(\pi \xi f T_s)] \quad (53)$$

$$c_2(p_s, \xi) = p_s^2(1 - p_s) \cos(\pi \xi f T_s) [2 \sin^2[\pi f T_s(1 - \xi)/2] - \sin^2(\pi \xi f T_s/2)] \quad (54)$$

$$c_3(p_s) = p_s(1 - p_s) [1 - p_s + (2p_s - 3) \cos(\pi f T_s/4)] \quad (55)$$

$$c_4(p, p_s, \xi) = 2pp_s(1 - p_s)(1 - 2p) \sin^2[\pi f T_s/2] \sin[3\pi f T_s \xi/2] \quad (56)$$

$$c_5(p_s, \xi) = p_s^2 [(1 - p_s) \sin(\pi f T_s \xi) \sin[\pi f T_s \xi/2] + \sin(\pi \xi f T_s) \sin(5\pi f T_s \xi/2) \{1 - \cos(\pi f T_s) \cos(\pi f T_s \xi)\}] \quad (57)$$

$$c_6(p, p_s, \xi) = -p_s [(1 - p)(1 - p_s) + 1/2 p_s^2] \cos[\pi f T_s(1 - \xi)/4] - pp_s(1 - pp_s) \cos[\pi f T_s(1 + \xi)/4] + p_s^2 [\cos(3\pi \xi f T_s) + (1 - p_s) \cos(2\pi \xi f T_s)] \quad (58)$$

$$c_7(p, p_s, \xi) = p_s [p_s^2 \sin^2[\pi f T_s(1 - \xi)/2] - 2(1 - p_s)(p^2 - 0.5p_s) \sin^2(\pi f T_s/2) \cos(\pi f T_s \xi) + p(1 - pp_s) \sin^2[\pi f T_s(1 + \xi)/2] \quad (59)$$

$$c_8(p, p_s, \xi) = p_s [p(1 - pp_s) + (1 - p_s)(1 - 2p)] \sin^2[\pi f T_s(1 - \xi)/2] + 2p_s(1 - p)(1 - p_s) \sin^2(\pi f T_s/2) \cos(\pi f T_s \xi) \quad (60)$$

$$H_1(m, p, p_s, \xi) = p_s^2 [\sin^2(m\pi \xi) [1 + 2h_1(m, \xi)]^2 + (1 - 2p)^2 \cos^2(m\pi \xi) [1 - 2h_1(m, \xi)]^2] \quad (61)$$

$$H_2(m, p, p_s, \xi) = \sin^2(2m\pi \xi) + (1 - 2p)^2 h_2^2(m, p_s, \xi) \quad (62)$$

$$H_3(m, p, p_s, \xi) = 2p_s [2 \sin^2(m\pi \xi) \cos(m\pi \xi) [1 + 2h_1(m, \xi)] + (1 - 2p)^2 \cos(m\pi \xi) [1 - 2h_1(m, \xi)] h_2(m, p_s, \xi)] \quad (63)$$

where the parameters $h_1(m, \xi)$ and $h_2(m, p_s, \xi)$ in Eqns (61)-(63) are defined as

$$h_1(m, \xi) = \begin{cases} \cos^2(m\pi\xi/2), & m \text{ odd} \\ \sin^2(m\pi\xi/2), & m \text{ even} \end{cases} \quad (64)$$

$$h_2(m, p_i, \xi) = (1 - p_i)(-1)^m \cos(2m\pi\xi) \quad (65)$$

If we let $p = p_i = 1/2$, then Eqns (50)-(52) reduce to the results for balanced data stream (or data stream with equiprobable symbols) presented in [4]. Furthermore, if we let $p = p_i = 1/2$ and $\xi = 0$, then Eqns (50)-(52) reduce to the well-known result for a perfect Bi- ϕ data stream [6].

Since $2B_f/R_s \ll 1$, the harmonic components in Eqn (97) do not cause interference to the carrier tracking and the variance of the carrier tracking phase error in the presence of unbalanced and asymmetric Bi- ϕ data stream becomes

$$\sigma^2 = 1/\rho_o + (\alpha/2) \tan^2(m_T) + (1/2)(2 - p_i)^2 \xi^2 \tan^2(m_i) \quad (66)$$

where

$$\alpha = \int_{-\infty}^{\infty} |H(2\pi f)|^2 S_{cBi-\phi}(f) df \quad (67)$$

The conditional probability of error for this case is the same as Eqns (36) and (39) except that the parameter $\lambda_k(i)$ is replaced by

$$\lambda_k(i) = \frac{\int_0^{T_s} g_{Bi-\phi}(t) g_{iBi-\phi}(t + kT_s) dt}{\int_0^{T_s} |g_{Bi-\phi}(t)|^2 dt}, \quad i = 1, 2, 3, 4 \quad (68)$$

where $g_{iBi-\phi}(t)$ is the output of an ideal bandpass filter corresponding to the input $P_{iBi-\phi}(t)$, for $i = 1, 2, 3, 4$, and $g_{Bi-\phi}(t)$ is the output of the ideal filter for a perfect data stream which is given by [Eqn (55), 1]. The output response $g_{iBi-\phi}(t + kT_s)$, for $i = 1, 2, 3, 4$, can easily be shown to be

$$g_{iBi-\phi}(t + kT_s) = \frac{1}{\pi} [\text{si}\{2\pi B(t + T_s(k - 1/2))\} - 2\text{si}\{2\pi B(t + T_s(k - \xi))\} \\ + \text{si}\{2\pi B(t + T_s(k - 1/2 - \xi))\}] \quad (69)$$

$$g_{2Bi-\phi}(t + kT_s) = - \frac{1}{\pi} [\text{si}\{2\pi B(t + T_s(k + 1/2))\} - 2\text{si}\{2\pi B(t + T_s(k + \xi))\} \\ + \text{si}\{2\pi B(t + T_s(k - 1/2 + \xi))\}] \quad (70)$$

$$g_{3Bi-\phi}(t + kT_s) = - \frac{1}{\pi} [\text{si}\{2\pi B(t + T_s(k + 1/2))\} - 2\text{si}\{2\pi B(t + T_s(k - \xi))\} \\ + \text{si}\{2\pi B(t + T_s(k - 1/2))\}] \quad (71)$$

$$g_{4Bi-\phi}(t + kT_s) = - \frac{1}{\pi} [\text{si}\{2\pi B(t + T_s(k + 1/2))\} - 2\text{si}\{2\pi B(t + T_s(k + \xi))\} \\ + \text{si}\{2\pi B(t + T_s(k - 1/2))\}] \quad (72)$$

Using the variance found in Eqn (66) and the parameter $\lambda_k(i)$ found in Eqn (68) the average error probability can be calculated as before and the results for the second order PLL [Eqn (24), 1] are shown in Figures 4-6. These figures plot the SER as a function of symbol SNR for $m_T = 1.25$ rad and $2B_1/R_s = 0,001$,

Figure 4 presents the SER performance for fixed BT_s of 3 and $\xi = 2\%$ with p is a parameter. This figure shows that the performance of PCM/PM/Bi-Phase is also sensitive to unbalanced data when there exists data asymmetry. The symbol SNR degradation is more than 1.2 dB when p deviates from 0.45.

Figure 5 shows the behavior of PCM/PM/Bi-Phase in the presence of data asymmetry. For fixed values of $BT_s = 3$, $p = 0.45$, the results shows that the SER performance is quite sensitive to data asymmetry, In [1] we have pointed out that in order to compare the results presented in Figure 5 with those in Figure 2 for NRZ data, use equal amounts of asymmetry as measured by the actual time displacement of both waveforms transitions. For a fair comparison, we replace ξ in Figure 5 by 2ξ when compared with Figure 2. As an example, the SER curve for PCM/PM/NRZ operating at 2% data asymmetry shown in Figure 2 corresponds to the 4% data asymmetry curve for PCM/PM/Bi- ϕ shown in Figure 5.

Figure 6 depicts the SER performance for PCM/PM/Bi-Phase for bandlimiting channel. Numerical results show that the performance is susceptible to bandlimiting channel. For $\xi = 2\%$ and $p = 0.45$, The degradation is unacceptable for $BT_s = 1$, and is about 0,5 dB or more when $BT_s = 3$.

S. Numerical Results and Discussions

Performance comparisons between PCM/PM/NRZ and PCM/PM/Bi- ϕ are illustrated in Figures 7-9 for uncoded systems. These figures plot the SER performance as a function of symbol SNR for $m_T = 1.25$ rad and $2B_T/R_s = 0.001$. Figure 7 compares the performance of PCM/PM/NRZ and PCM/PM/Bi- ϕ in the presence unbalanced data, For a fixed data asymmetry of 2 %, this figure shows that both PCM/PM/NRZ and PCM/PM/Bi- ϕ experience unacceptable degradations when the probability of transmitting a + 1 pulse deviates from 0.5, and that PCM/PM/NRZ is more sensitive to unbalanced data than PCM/PM/Bi- ϕ .

Figure 8 shows the performance comparison for both systems with data asymmetry as a parameter. As the data asymmetry increases from 0 to 2 % the SER performance of PCM/PM/Bi- ϕ degrades seriously. For $p = 0.45$, $BT_s = 3$, $\xi = 2$ % and $SER \leq 10^{-4}$, the degradation in symbol SNR for PCM/PM/Bi- ϕ is about 1.5 dB or more, and less than 0.5 dB for PCM/PM/NRZ.

Figure 9 compares the SER performance for both systems for fixed data asymmetry of 2 % and $p = 0.45$ with BT_s (bandwidth-to-data rate ratio) as a parameter. For $BT_s = 1$, the symbol SNR degradation for PCM/PM/Bi- ϕ is unacceptable. On the other hand, under the same operating conditions, the symbol SNR degradation for PCM/PM/NRZ is less than 1 dB for $SER \leq 10^{-4}$. For $BT_s = 3$, the symbol SNR degradations for PCM/PM/NRZ and PCM/PM/Bi- ϕ are at the order of 0.2 dB and 0.8 dB or less for $SER \geq 10^{-6}$, respectively. This figure also shows that PCM/PM/Bi- ϕ is more susceptible to bandwidth constraint than PCM/PM/NRZ.

Since the international Consultative Committee for Space Data System (CCSDS) recommends the convolutional coding scheme with rate 1/2 constraint length 7 for space telemetry signal, we are interested to determine the symbol SNR degradation for coded PCM/PM/NRZ and PCM/PM/Bi- ϕ systems due to the presence of both imperfect data stream and ISI. As mentioned earlier in [3], an exact analysis for coded systems is not possible. [3] suggested that the symbol SNR degradation due to imperfect data stream and ISI can be estimated from the decoder bit error performance curve and the results shown Figures 1-6 and the ideal bit error performance curve by assuming that the coding is transparent to the imperfect data stream and ISI and using the uncoded energy-to-noise density ratios corresponding to the coded bit energy-to-noise density ratios at the desired bit error rates. Tables 1 and 2 show the symbol energy-to-noise density ratio (E_s/N_o) degradations in dB for PCM/PM/NRZ and PCM/PM/Bi- ϕ , respectively. The numerical results presented in these tables are for $m_T = 1.25$ rad, $2B_T/R_s = 0.001$, $\xi = 2$ % and $p = 0.45$. The selected values of E_s/N_o presented in these tables are 0.8, 1.5 and 1.95 dB. The selected values correspond to bit energy-to-noise density ratios $E_b/N_o = 3.8, 4.5$, and 4.95 dB which correspond to Viterbi decoder bit error probabilities $P_b = 10^{-4}, 10^{-5}$ and 10^{-6} , for rate 1/2 and constraint length 7, respectively,

Table 1. Degradation Due to Imperfect Data and ISI for Rate 1/2 Convolutionally Encoded Random NRZ Data, $m_T = 1,25$ rad, $2B_I/R_s = 0.001$, $p = 0.45$, $\xi = 2$ %.

SER	E_s/N_o , dB	Symbol SNR Degradation, A(dB)		
		$BT_s = 1$	$BT_s = 2$	$BT_s = 3$
10^{-4}	0.80	0.30	0.05	0.01
10^{-5}	1.50	0.37	0.08	0,06
10^{-6}	1.95	0.47	0.10	0,07

Table 2. Degradation Due to Imperfect Data and ISI for Rate 1/2 Convolutionally Encoded Random Bi- ϕ Data, $m_T = 1,25$ rad, $2B_I/R_s = 0.001$, $p = 0.45$, $\xi = 290$.

SER	E_s/N_o , dB	Symbol SNR Degradation, Δ (dB)		
		$BT_s = 1$	$BT_s = 2$	$BT_s = 3$
10^{-4}	0.80	2.53	0.4	0.25
10^{-5}	1.50	2.95	0.8	0.55
10^{-6}	1.95	3.33	0.9	0.63

6. Conclusion

Combined effects of the imperfect data stream and ISI caused by the bandlimiting channel on the performance of PCM/PM communications systems are investigated in this paper. Analytical models to predict the SER performance for both uncoded PCM/PM/NRZ and PCM/PM/Bi- ϕ systems were derived. In addition, the symbol SNR degradations for rate 1/2 constraint length 7 convolutional coded systems are also evaluated.

Numerical results show that theoretical predictions for the total symbol SNR degradation of the receivers due to the presence of both imperfect data and ISI are not the algebraic sum of each symbol SNR degradation due to a single source of degradation found in [1]. The results also show that, for 2 % data asymmetry, both PCM/PM/NRZ and PCM/PM/Bi- ϕ are susceptible to unbalanced data stream, and that PCM/PM/NRZ is more sensitive to unbalanced data than PCM/PM/Bi- ϕ . On the other hand, PCM/PM/Bi- ϕ is more susceptible to data asymmetry while PCM/PM/NRZ is not.

Furthermore, the results show that, for $BT_s = 3$, the SER performance of PCM/PM/NRZ is acceptable for both near earth and deep space missions. However, for PCM/PM/Bi- ϕ , the SER performance is found to be unacceptable for deep space missions and may be acceptable for near earth missions.

Acknowledgement

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Appendix

The conditional probability of error for $M = 2$

$$\begin{aligned}
 P_e(\theta_e) = & p[p_3p_3p_3p_3\text{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{.2}(3) + \lambda_{.1}(3) + \lambda_1(3) + \lambda_2(3))\cos\theta_e\} \\
 & - t \quad p_4p_2p_1p_4\text{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{.2}(4) - \lambda_{.1}(2) - \lambda_1(1) - \lambda_2(4))\cos\theta_e\} \\
 & + \quad p_2p_3p_3p_1\text{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{.2}(2) + \lambda_{.1}(3) + \lambda_1(3) - \lambda_2(1))\cos\theta_e\} \\
 & + \quad p_4p_2p_3p_1\text{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{.2}(4) - \lambda_{.1}(2) + \lambda_1(3) - \lambda_2(1))\cos\theta_e\} \\
 & + \quad p_3p_3p_3p_1\text{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{.2}(3) + \lambda_{.1}(3) + \lambda_1(3) - \lambda_2(1))\cos\theta_e\} \\
 & + \quad p_3p_3p_1p_2\text{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{.2}(3) + \lambda_{.1}(3) - \lambda_1(1) + \lambda_2(2))\cos\theta_e\} \\
 & + \quad p_1p_2p_3p_3\text{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{.2}(2) - \lambda_{.1}(3) + \lambda_1(3) + \lambda_2(1))\cos\theta_e\} \\
 & - t \quad p_2p_3p_3p_3\text{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{.2}(2) + \lambda_{.1}(3) + \lambda_1(3) + \lambda_2(3))\cos\theta_e\} \\
 & + \quad p_3p_3p_1p_4\text{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{.2}(3) + \lambda_{.1}(3) - \lambda_1(1) - \lambda_2(4))\cos\theta_e\} \\
 & + \quad p_1p_2p_1p_2\text{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{.2}(1) - \lambda_{.1}(2) - \lambda_1(1) + \lambda_2(2))\cos\theta_e\} \\
 & + \quad p_1p_2p_3p_1\text{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{.2}(1) - \lambda_{.1}(2) + \lambda_1(3) - \lambda_2(1))\cos\theta_e\} \\
 & + \quad p_4p_2p_3p_3\text{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{.2}(4) - \lambda_{.1}(2) + \lambda_1(3) + \lambda_2(3))\cos\theta_e\} \\
 & + \quad p_2p_3p_1p_2\text{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{.2}(2) + \lambda_{.1}(3) - \lambda_1(1) + \lambda_2(2))\cos\theta_e\} \\
 & + \quad p_1p_2p_1p_4\text{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{.2}(1) - \lambda_{.1}(2) - \lambda_1(1) - \lambda_2(4))\cos\theta_e\} \\
 & + \quad p_2p_3p_1p_4\text{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{.2}(2) + \lambda_{.1}(3) - \lambda_1(1) - \lambda_2(4))\cos\theta_e\} \\
 & + \quad p_4p_2p_1p_2\text{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{.2}(4) - \lambda_{.1}(2) - \lambda_1(1) + \lambda_2(2))\cos\theta_e\} \\
 & + \quad q[p_3p_1p_2p_3\text{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{.2}(3) - \lambda_{.1}(1) - \lambda_1(2) - \lambda_2(3))\cos\theta_e\} \\
 & + \quad p_4p_4p_4p_4\text{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{.2}(4) + \lambda_{.1}(4) + \lambda_1(4) + \lambda_2(4))\cos\theta_e\}
 \end{aligned}$$

$$\begin{aligned}
& + p_2 p_1 p_2 p_1 \operatorname{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{.2}(2) - \lambda_{.1}(1) - \lambda_1(2) + \lambda_2(1))\cos\theta_e\} \\
& + p_4 p_4 p_2 p_1 \operatorname{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{.2}(4) + \lambda_{.1}(4) - \lambda_1(2) + \lambda_2(1))\cos\theta_e\} \\
& + p_3 p_1 p_2 p_1 \operatorname{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{.2}(3) - \lambda_{.1}(1) - \lambda_1(2) + \lambda_2(1))\cos\theta_e\} \\
& + p_3 p_1 p_4 p_2 \operatorname{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{.2}(3) - \lambda_{.1}(1) + \lambda_1(4) - \lambda_2(2))\cos\theta_e\} \\
& + p_1 p_4 p_2 p_3 \operatorname{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{.2}(2) + \lambda_{.1}(4) - \lambda_1(2) - \lambda_2(1))\cos\theta_e\} \\
& + p_2 p_1 p_2 p_3 \operatorname{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{.2}(2) - \lambda_{.1}(1) - \lambda_1(2) - \lambda_2(3))\cos\theta_e\} \\
& + p_3 p_1 p_4 p_4 \operatorname{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{.2}(3) - \lambda_{.1}(1) + \lambda_1(4) + \lambda_2(4))\cos\theta_e\} \\
& + p_1 p_4 p_4 p_2 \operatorname{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{.2}(1) + \lambda_{.1}(4) + \lambda_1(4) - \lambda_2(2))\cos\theta_e\} \\
& + p_1 p_4 p_2 p_1 \operatorname{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{.2}(1) + \lambda_{.1}(4) - \lambda_1(2) + \lambda_2(1))\cos\theta_e\} \\
& + p_4 p_4 p_2 p_3 \operatorname{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{.2}(4) + \lambda_{.1}(4) - \lambda_1(2) - \lambda_2(3))\cos\theta_e\} \\
& + p_2 p_1 p_4 p_2 \operatorname{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{.2}(2) - \lambda_{.1}(1) + \lambda_1(4) - \lambda_2(2))\cos\theta_e\} \\
& + p_1 p_4 p_4 p_4 \operatorname{erfc}\{\sqrt{E_s/N_o}(1 - \lambda_{.2}(1) + \lambda_{.1}(4) + \lambda_1(4) + \lambda_2(4))\cos\theta_e\} \\
& + p_2 p_1 p_4 p_4 \operatorname{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{.2}(2) - \lambda_{.1}(1) + \lambda_1(4) + \lambda_2(4))\cos\theta_e\} \\
& + p_4 p_4 p_4 p_2 \operatorname{erfc}\{\sqrt{E_s/N_o}(1 + \lambda_{.2}(4) + \lambda_{.1}(4) + \lambda_1(4) - \lambda_2(2))\cos\theta_e\}]
\end{aligned}$$

Figure 1. SER for Various Probabilities of Mark

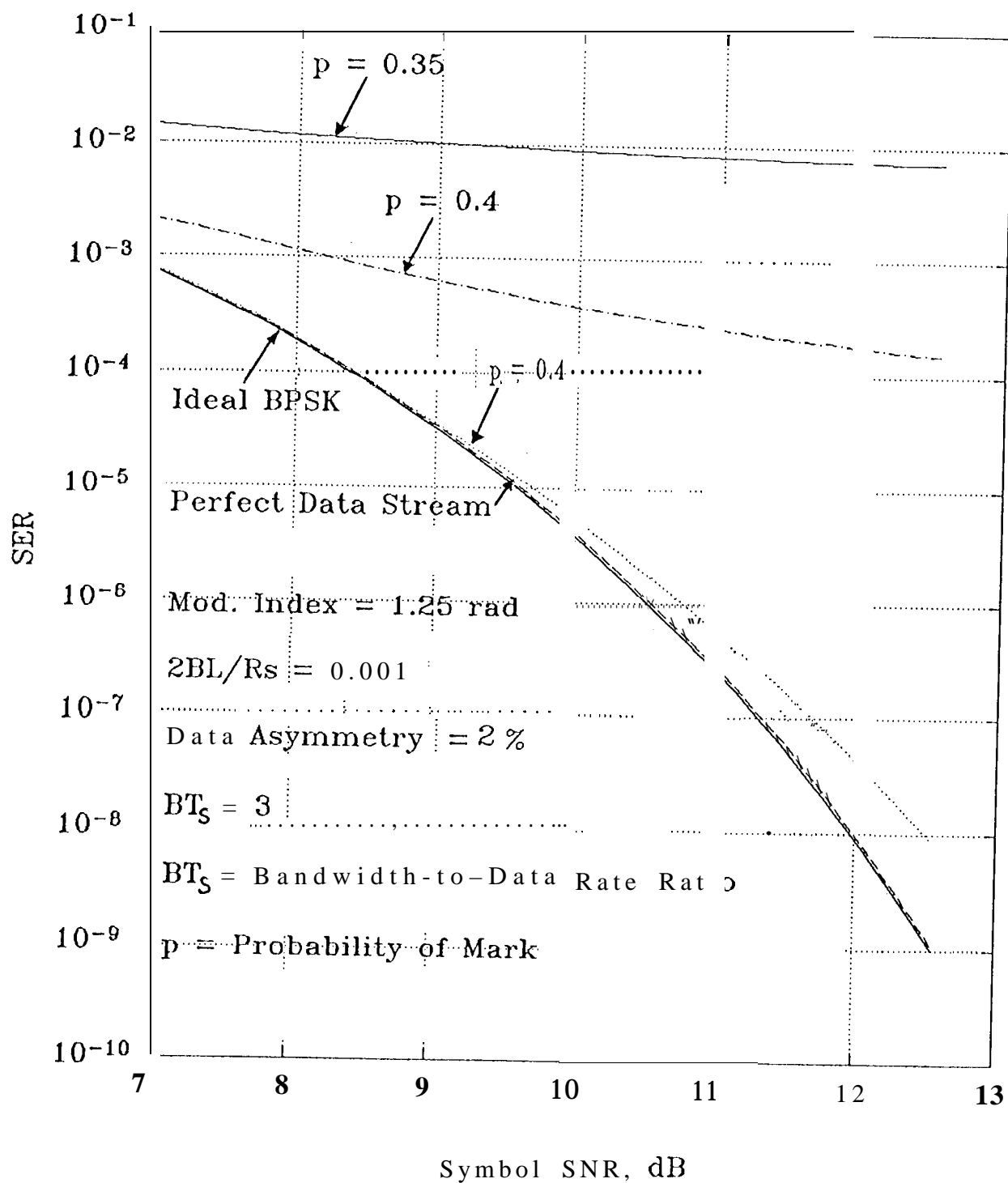


Figure 2. SER for Various Values of Data Asymmetry

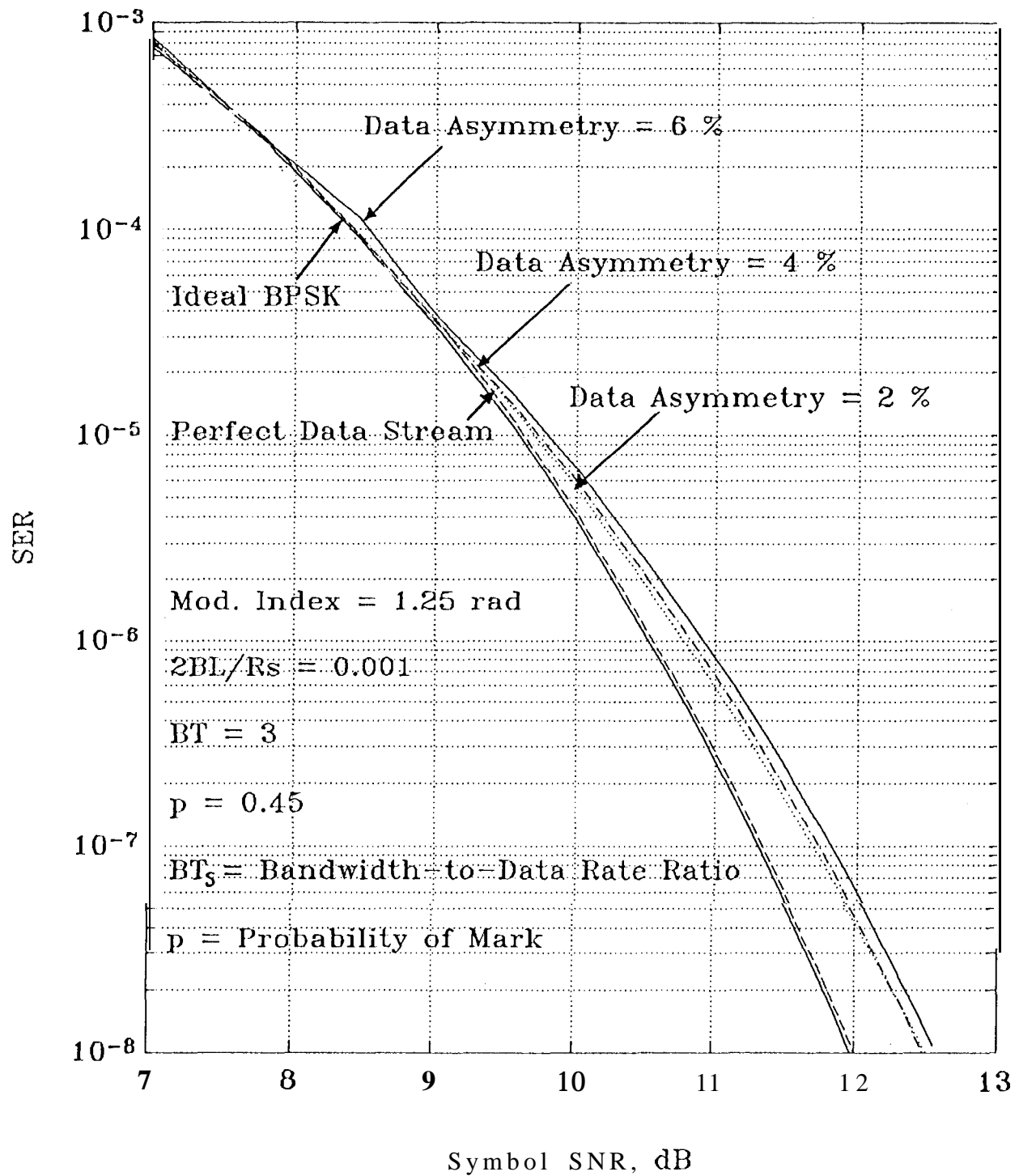


Figure 3. PCM/PM/NRZ-SER for Various Values of BT

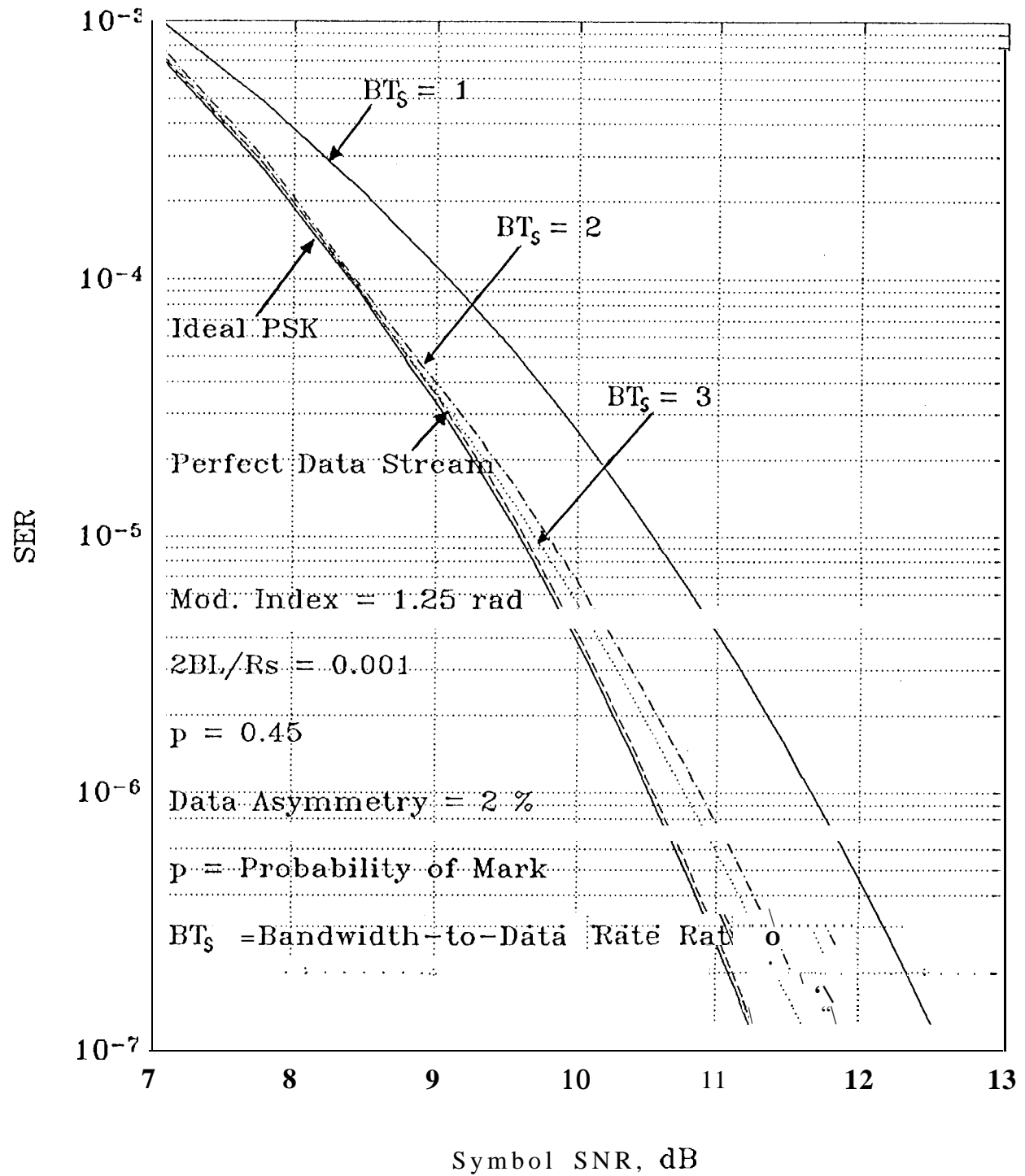


Figure 4. Bi-Phase-SER for Various Values of p

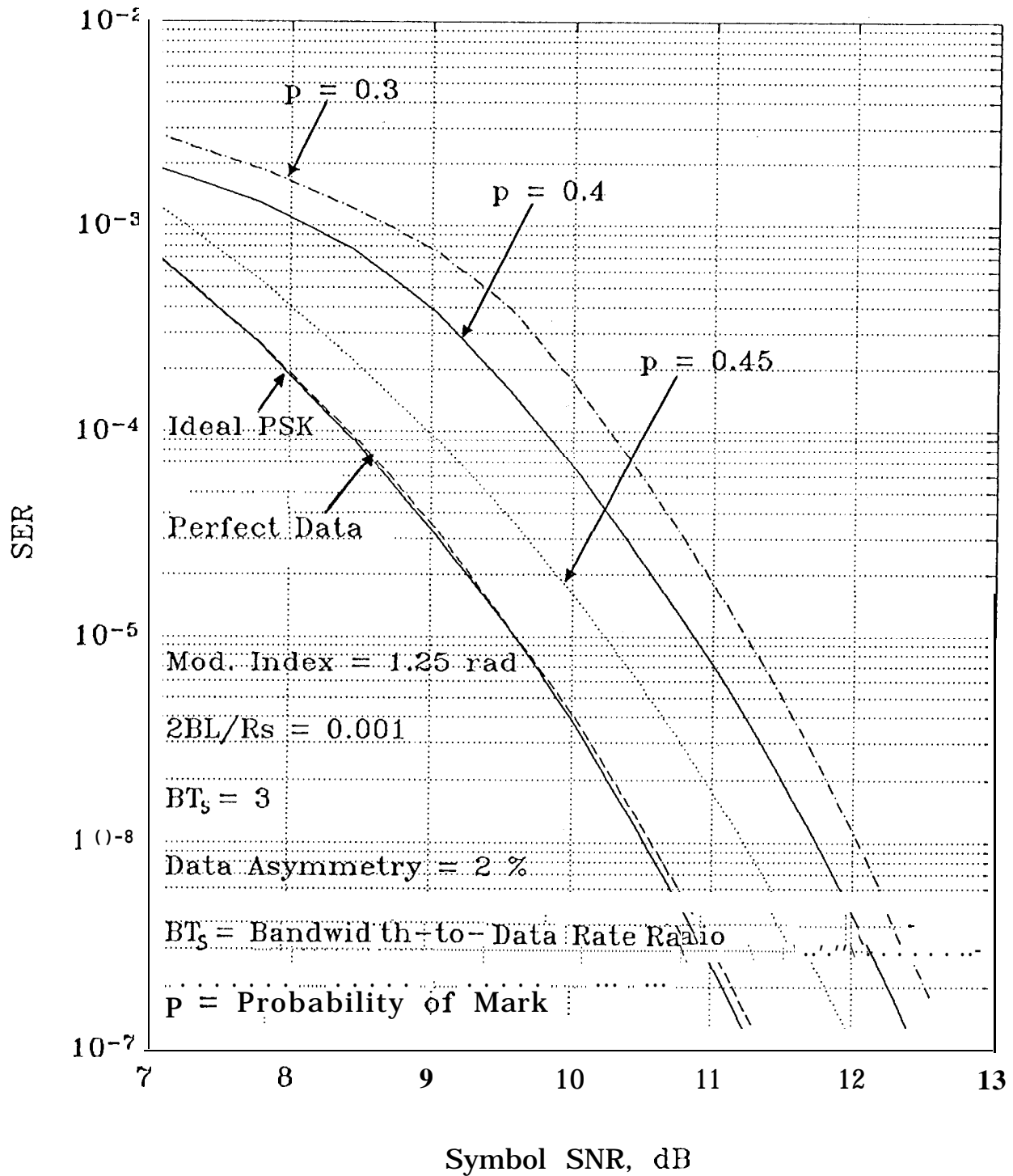


Figure 5. Bi-Phase-SER With Data Asymmetry

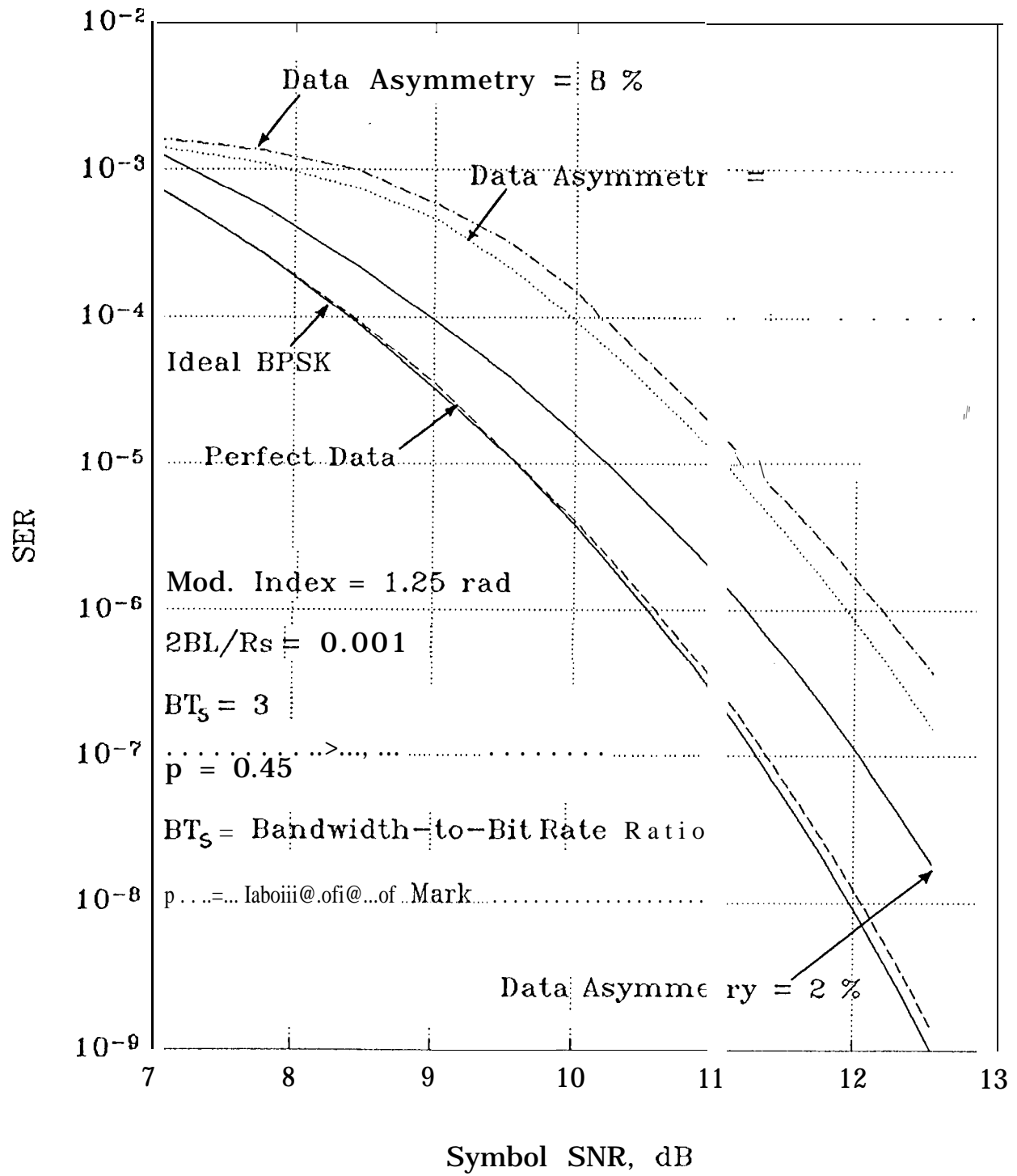


Figure 6. Bi-Phase-SER for Various Values of BT

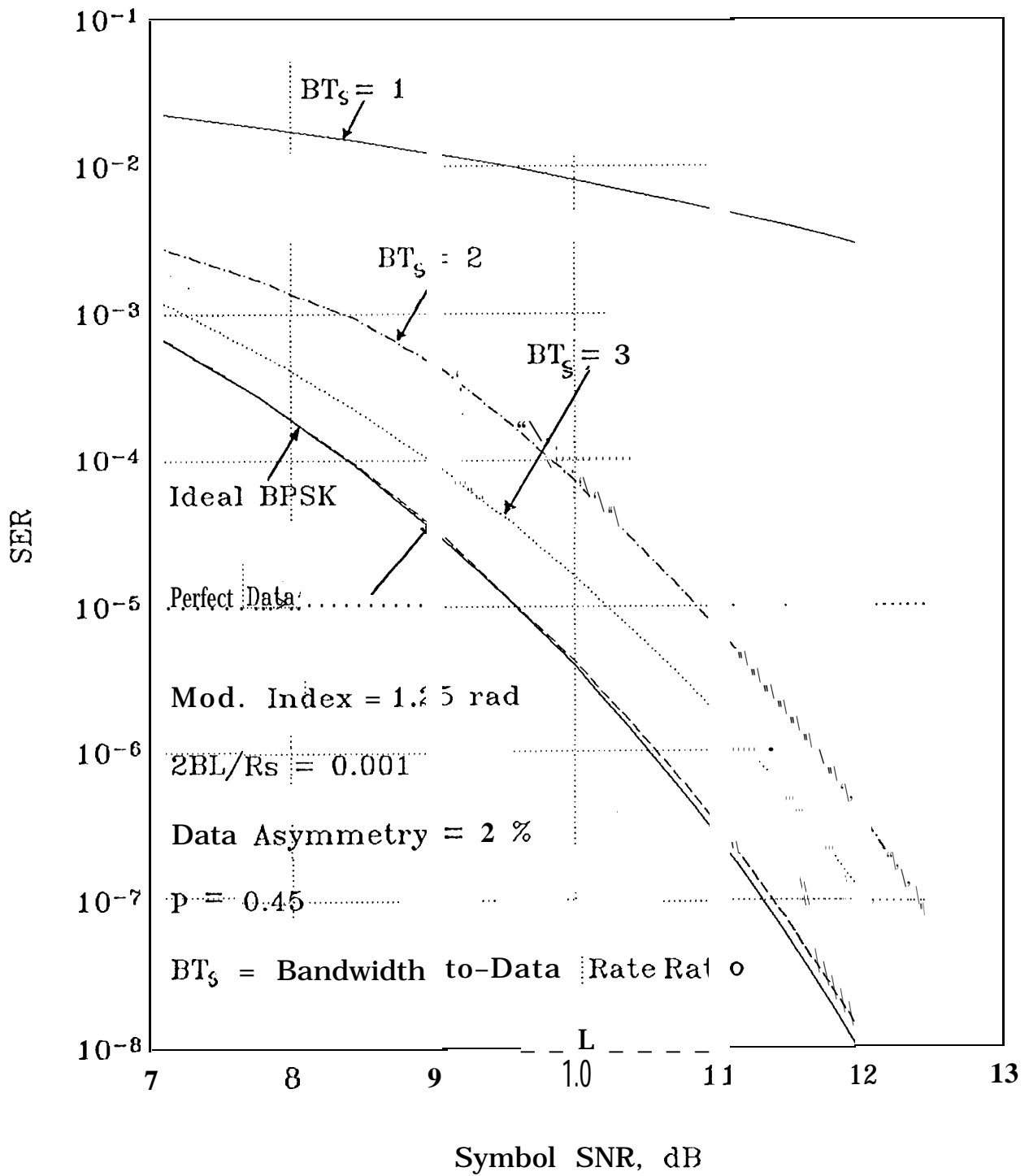


Figure i'. Performance Comparison for Unbalanced Data

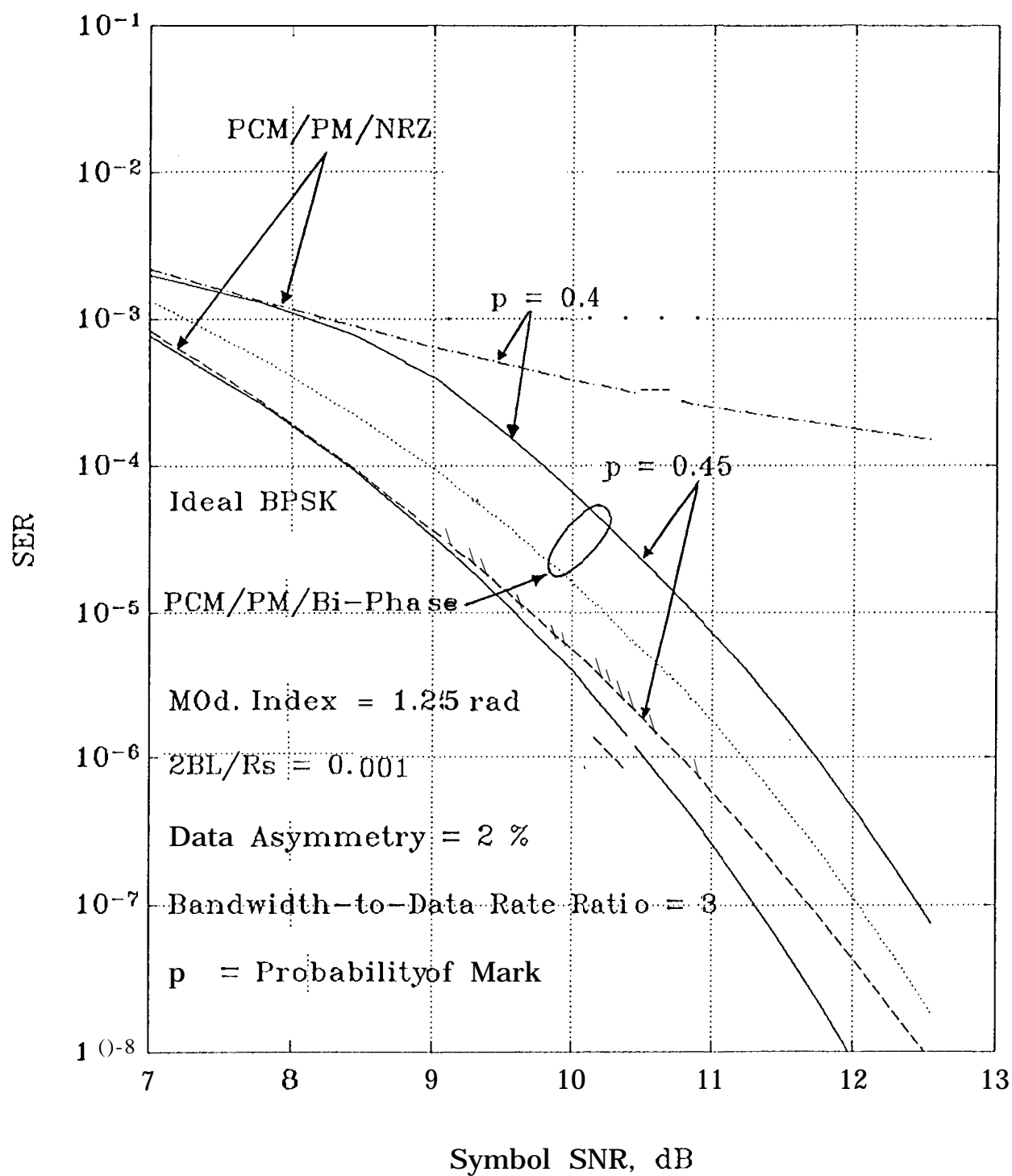


Figure 9. Performance Comparison for Bandlimiting Channel

